

Cherenkov radiation and pair production by particles traversing laser beams

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Abstract

It is shown that Cherenkov radiation can be observed at TESLA in electron collisions with optical laser pulses. The prospects for it to be observed at SLC, LEP, LHC and RHIC are discussed. The conclusions are compared with results for pair production.

The problem of very high energy charged particles collisions with laser beams is widely discussed now, mostly in connection with the e^+e^- -pair production (see latest references [1, 2, 3, 4]) at SLC and TESLA X-ray laser facilities [5, 6, 7] and with some other issues of fundamental physics.

Here, I would like to note that optical lasers can be used for studies of Cherenkov radiation. The results can be applied to measurements of beam energy, laser bunch parameters and, in general, to verify our ideas about the properties of the "photon medium".

X-ray lasers have been proposed for use in e^+e^- -pair production studies because the quanta energies are high enough to reach the threshold energy which in c.m.s. is equal to $m + 2m_e$, where m_e is the electron mass and m is the accelerated particle mass (equal to m_e at SLC, LEP, TESLA, to the proton mass at LHC and to the nucleus mass at RHIC). At the same time, for studies of Cherenkov radiation another characteristics of a laser, namely its peak power density S , is important. It determines the index of refraction n of the "photon medium" in laser bunches. The difference of n from 1 is proportional to the density of photons in a laser pulse, i.e., to S . Just this difference defines the threshold of Cherenkov radiation, its emission angle and intensity¹. The parameter S is higher for optical lasers than for presently available X-ray lasers. That is why they would be preferred for Cherenkov radiation studies nowadays. Besides, the energy limitations also favour optical lasers for this purpose.

The necessary conditions for Cherenkov radiation to be observed are the excess of the index of refraction n over 1, i.e.

$$\Delta n = n - 1 > 0 \tag{1}$$

and the real emission angle, given by the formula

$$\cos \theta = \frac{1}{\beta n}, \tag{2}$$

¹The analogous problem was considered in Ref. [8] for particles traversing the cosmic microwave background radiation. The density of relic photons is, however, extremely low and, therefore, the index of refraction is so close to 1 that the threshold energy is too high for this effect to be observable.

where $\beta = v/c = \sqrt{1 - \frac{m^2}{E^2}}$, m, E are the particle mass and energy. For small values of m/E and Δn one gets

$$\theta \approx \sqrt{2\Delta n - \frac{m^2}{E^2}} = \sqrt{2\Delta n - \gamma^{-2}}. \quad (3)$$

Hence, the condition for the energy to exceed the threshold for Cherenkov radiation E_{Ct} is written as

$$\gamma m = E \geq E_{Ct} = \frac{m}{\sqrt{2\Delta n}} = \gamma_{Ct} m. \quad (4)$$

It is easily seen that the threshold can become very high for small Δn .

The formula (3) can be rewritten as

$$0 \leq \theta = \frac{\sqrt{\gamma^2 - \gamma_{Ct}^2}}{\gamma \gamma_{Ct}} \leq \frac{1}{\gamma_{Ct}} = \theta_{max} \quad (\gamma \rightarrow \infty). \quad (5)$$

It is seen that the emission angles of Cherenkov radiation increase from 0 at the threshold to $\theta_{max} = \gamma_{Ct}^{-1}$ for $\gamma \rightarrow \infty$. However, already at $E = 2E_{Ct}$ this angle is very close to θ_{max} ($\theta(2E_{Ct}) \approx 0.866 \theta_{max}$).

The number of Cherenkov photons emitted by a single particle with the electric charge e in the interval of frequencies $d\omega$ from the path length dl is given by the common expression [9]

$$\frac{dN_1}{d\omega dl} = 2\alpha \Delta n, \quad (6)$$

where the fine structure constant $\alpha = e^2 \approx 1/137$. Thus all physical characteristics of the process are determined by the value Δn . The intensity of the radiation (6) decreases with the threshold energy (4) increase:

$$\frac{dN_1}{d\omega dl} = \frac{\alpha m^2}{E_{Ct}^2} = \frac{\alpha}{\gamma_{Ct}^2}. \quad (7)$$

The value of Δn is uniquely related to the polarization operator of $\gamma - \gamma$ scattering. It has been calculated in Ref. [10]. According to these results, the value of Δn is positive for energies below or nearby the threshold for e^+e^- -pair production. It stays almost constant at low energies and increases by a factor of about 2.5 at the threshold for e^+e^- -pair production. At higher energies it decreases and becomes negative so that Cherenkov radiation is impossible there. Even though it can again become positive at extreme energies where the hadronic channels are important, this region is completely inaccessible in collisions with laser beams.

For energies much below the pair production threshold, one can estimate:

$$\Delta n \approx \frac{14(4\pi\alpha)^2 S}{45\pi m_e^4}. \quad (8)$$

This expression follows from the formula (31) in Ref. [10] (see also Fig. 2 there) if the variable k^{-1} there is written in notations adopted in this paper. Thus, the peak power density S defines main features of Cherenkov radiation. The Cherenkov threshold does not depend on laser energy at the same values of S . Let us note, that S itself is, however, related to the photon energy as

$$S = \nu\omega_L, \quad (9)$$

where ν is the photon density in a laser bunch. The value of γ_{Ct} is completely determined by the laser peak power density as seen from formulas (4), (8). It is the same for electrons and protons. Therefore, the values of E_{Ct} are approximately 2000 times higher for protons than for electrons. The formulas (3) and (8) show that by measuring the angle θ one can get the energy of the particle beam and the peak power density of the laser.

In comparison, the energy threshold for the e^+e^- -pair production processes in high energy head-on collisions is given by

$$E_{pt} \approx \frac{m_e(m + m_e)}{\omega_L}. \quad (10)$$

It depends on the energy of laser quanta ω_L and is much lower for X-ray lasers than for optical lasers. It is approximately 1000 times higher for protons than for electrons.

The condition for Cherenkov radiation threshold to be below the pair production threshold imposes the restriction on the laser quanta energies:

$$\omega_L < \sqrt{2\Delta n}m_e(1 + \frac{m_e}{m}) = \gamma_{Ct}^{-1}m_e(1 + \frac{m_e}{m}). \quad (11)$$

The condition (11) differs for electron and proton beams only by a factor about 2 in the right hand side.

For optical and X-ray lasers, according to [2], we accept, correspondingly, $\omega_L = 1.2$ eV (actually, it varies from 0.12 eV for CO₂-laser to 1.8 eV for ruby laser) and 3.1 keV, peak power densities $S = 3 \cdot 10^{22}$ W/cm² = $5 \cdot 10^{16}$ eV⁴ and $8 \cdot 10^{19}$ W/cm² (with a possible goal $7 \cdot 10^{29}$ W/cm²).

Using these characteristics, one concludes that the condition (11) is satisfied for optical lasers with $S > 10^{21}$ W/cm² and not valid for the presently available X-ray lasers. To satisfy it for X-ray lasers, one must achieve the peak power density as high as $3 \cdot 10^{26}$ W/cm² which is, nevertheless, within the proclaimed goals. Thus, in what follows, we discuss only optical lasers briefly referring to X-ray lasers for some estimates.

The numerical value of the index of refraction for optical laser quanta is given by

$$\Delta n = 8.4 \cdot 10^{-4} \frac{S}{m_e^4} = 6.2 \cdot 10^{-10}. \quad (12)$$

Therefore, the typical angles and threshold γ -factors for Cherenkov radiation are

$$\theta_{max} = 3.5 \cdot 10^{-5}; \quad \gamma_{Ct} = 2.8 \cdot 10^4. \quad (13)$$

It implies that the energy threshold for Cherenkov radiation is exceeded at SLC, LEP, TESLA since it is $E_{Ct}^{(e)} = 14$ GeV.

The pair production threshold for optical lasers is about 440 GeV. Thus SLC and LEP energies are well below it while TESLA is just close² to the threshold value.

For proton beams the Cherenkov radiation threshold $E_{Ct}^{(p)} = 26$ TeV is too high even for LHC. If the optical lasers with the peak power density $S > 5 \cdot 10^{23}$ W/cm² will become accessible, one can hope to observe this effect there as well. This energy is much lower than the threshold for pair production at proton accelerators which is about 400 TeV.

What concerns the X-ray laser facilities, the threshold for pair production (10) is well below the energies accessible at all high energy accelerators. However, it implies that we reach the region where Δn becomes negative (see [10]) and, therefore, there is no Cherenkov radiation unless the hadronic channels change this conclusion.

Now, let us calculate the intensity of the Cherenkov radiation for electron beams and compare it with the main background process of Compton scattering³. The total number of Cherenkov quanta emitted in the energy interval $d\omega$ per unit interval of time by a bunch containing n_e electrons which collide with laser bunches of the coherent spike length l with frequency f is

$$\frac{dN_{Ch}}{d\omega} = f n_e \frac{dN_1}{d\omega} = f n_e \nu l (4\pi\alpha)^3 \frac{7\omega_L}{45\pi^2 m_e^4}. \quad (14)$$

Here we used formulas (6), (8) and (9).

The corresponding formula for the Compton scattering process looks like

$$\frac{dN_{\gamma e}}{d\omega} = L \frac{d\sigma_{\gamma e}}{d\omega} = f \frac{n_e n_\gamma}{4\pi R^2} \frac{d\sigma_{\gamma e}}{d\omega} = 0.25 f n_e \nu l \frac{d\sigma_{\gamma e}}{d\omega}. \quad (15)$$

Here L is the luminosity, R and l are the radius and the coherent spike length of the laser bunch, so that the number of photons n_γ in the bunch is related to its density ν as $n_\gamma = \pi R^2 l \nu$, $d\sigma_{\gamma e}/d\omega$ is the Compton scattering cross section.

The ratio of the intensities of the two processes is

$$\frac{dN_{Ch}}{dN_{\gamma e}} = \frac{7 \cdot 2^8 \pi \alpha^3 \omega_L}{45 m_e^4 d\sigma_{\gamma e}/d\omega}. \quad (16)$$

The kinematics of the Compton scattering in the laboratory system leads to the following relation between the emission angle θ and energy ω of the scattered quanta:

$$\cos \theta = \left(1 + \frac{\omega_L}{E} - \frac{2\omega_L}{\omega}\right) \left(1 - \frac{\omega_L}{E}\right)^{-1} \approx 1 - \frac{2\omega_L}{\omega}. \quad (17)$$

²However, notice the rather wide spread of available wavelengths for optical lasers mentioned above.

³The final results are valid for any charged particles.

The precise limits imposed by this relation on the energy of emitted quanta are given by $\omega_L \leq \omega \leq E$. In the right hand side of (17) we have taken into account that the particle beam energy is much higher than the photons energies $E \gg \omega \geq \omega_L$. At the angles typical for Cherenkov radiation (13) the energies of the scattered quanta $\omega \approx 4$ GeV satisfy these inequalities.

The expression for the γe -scattering cross section [11] is written as

$$\frac{d\sigma_{\gamma e}}{d\omega} = 8\pi r_0^2 \frac{U_0}{m_e^2 \kappa_1^2} \omega_L \left(1 - \frac{\omega_L}{E}\right)^{-1}, \quad (18)$$

where

$$U_0 = 4 \left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right)^2 - 4 \left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right) - \frac{\kappa_1}{\kappa_2} - \frac{\kappa_2}{\kappa_1}, \quad (19)$$

and

$$m_e^2 \kappa_1 = -2pk_L \approx -4E\omega_L; \quad (20)$$

$$m_e^2 \kappa_2 = 2pk \approx 2E\omega(1 - \cos \theta) = 4E\omega_L \frac{E - \omega}{E - \omega_L}. \quad (21)$$

For $E \gg \omega \gg \omega_L$, one gets $\kappa_1 \approx -\kappa_2$, $U_0 \approx 2$ and

$$\frac{d\sigma_{\gamma e}}{d\omega} = \frac{\pi \alpha^2}{E^2 \omega_L}. \quad (22)$$

Finally, the ratio of Cherenkov to Compton effects is

$$\frac{dN_{Ch}}{dN_{\gamma e}} = \frac{7}{45} 2^8 \alpha \left(\frac{E\omega_L}{m_e^2} \right)^2 \approx 0.3 \left(\frac{E\omega_L}{m_e^2} \right)^2. \quad (23)$$

This ratio depends only on the c.m.s. energy and fastly increases with the laboratory energies of both particle and laser beams. It is of the order of 1 near the e^+e^- -pair production threshold⁴ (10) for electron beams. As expected, it does not depend on beam properties.

At TESLA energies, Cherenkov radiation is about 5 times stronger than the Compton effect for optical laser photons. Its absolute intensity can be evaluated according to the formulas (6), (14). For the coherent spike length $dl \sim 0.3$ cm (the average value from Ref. [2]) and the energy interval for Cherenkov quanta $d\omega \sim 2m_e \sim 1$ MeV one estimates the number of quanta per unit time interval as

$$N_{Ch} \approx 0.15 f n_e. \quad (24)$$

It looks quite feasible and depends on the electron beam intensity $f n_e$ which will be available at TESLA.

At SLC and LEP energies with the adopted parameters of optical lasers we are well below the pair production threshold, and Cherenkov radiation is only about several per cents of the Compton process. Its share increases

⁴Moreover, the value of Δn increases somewhat when this threshold is approached (see [10]), and the factor $14/45\pi \approx 0.1$ should be replaced in (23) by 0.25 at the threshold.

with increase of ω_L . However, the use of higher energy lasers inevitably leads to the change of corresponding values of S and/or ν and, consequently, of the threshold energy E_{Ct} . Therefore, one should be careful in estimates. The practical feasibility of observation of such an effect at these energies should be considered in close relation to the definite conditions of a particular experiment. The optimum choice would be the laser with slightly higher energy and higher peak power density compared with parameters used in above estimates. This would diminish the role of the background and increase the emission angle.

At the end, let us note that, in principle, the heavy ion accelerator RHIC can be also used for pair production studies with X-ray lasers because the threshold energy according to (10) is equal to 165 GeV per nucleon that is available there. The Cherenkov radiation threshold is the same as for proton accelerators if estimated per nucleon. It has been discussed above in connection with LHC and is not reachable at RHIC.

I am grateful to V.A Maishev and A.I. Nikishov for discussions and comments.

This work is supported by the RFBR grant 00-02-16101.

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